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ON THE M/M/2/∞ RETRIAL QUEUE WITH LIMITED NUMBER OF RETRIALS

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In this paper retrial queue of the $M/M/2/\infty$ *- type in which the rate of input flow depends on the number of sources of retrial calls and in which the number of retrials of each customer is limited by a single attempt is considered.*

Key words: RETRIAL QUEUE, STATIONARY PROBABILITIES, CONTINUED FRACTIONS

The main characteristic of retrial queue is that a primary customer who finds busy the service facility upon arrival moment immediately leaves the servers area, but some time later he repeats his demand (see, for example, [1], [2]). Between retrials he is said to be in 'orbit'. Such queueing system with repeated attempts is used in designing local area networks, communication networks.

In this work we consider retrial queue of the $M/M/2/\infty$ - type in which the number of retrials of each customer is limited by a single attempt. That is, if a customer fails to enter the server facility at the retrial attempt, then the customer leaves the system without service.

A retrial queue with two servers is given by three parameters: λ_j, $j = 0,1,...$ is the rate of input flow depends on the number of sources for repeated attempt, μ is the service rate and ν is the rate of repeated attempt.

The system state at time t can be described by means of a process $Q(t) = (Q_0(t), Q_1(t))$, where $Q_0(t)$ is the number of busy servers at time t, $Q_1(t)$ is the number of sources for repeated attempt at time t. The process $Q(t)$ is a continuous time Markov chain with the state set $S = \{0,1,2\} \times Z_+$.

For the system condition in which a stationary regime exists is found.

Moreover, for calculation of the stationary probabilities effective scheme with use of the continued fractions is proposed.

Theorem 1. If the condition $\overline{\lim_{n\to\infty}} n^{-1}\lambda_n < \nu$ holds then stationary probabilities π_{ij}, $(i, j) \in S$ of the process $Q(t)$ can be represented in the following form:

$$\pi_{0j} = \left(\prod_{k=0}^{j-1} x_k\right)\pi_{00},\ \pi_{1j} = \frac{\lambda_j + j\nu}{\mu}\left(\prod_{k=0}^{j-1} x_k\right)\pi_{00},\ j = 0,1,\ldots,$$

$$\pi_{2j} = \frac{1}{2\mu^2}((\lambda_j + j\nu)^2 + j\nu\mu - (j+1)\nu\mu x_j)\left(\prod_{k=0}^{j-1} x_k\right)\pi_{00},\ j = 0,1,\ldots,$$

where

$$(\pi_{00})^{-1} = \frac{1}{2\mu^2}\left(\sum_{j=0}^{\infty}\left((\lambda_j + \mu + j\nu)^2 + \mu(\mu + j\nu - (j+1)\nu x_j)\right)\left(\prod_{k=0}^{j-1} x_k\right)\right)$$

$$x_j = \left[0; \frac{\gamma_{j+1}}{\beta_{j+1}}; \frac{\gamma_{j+2}}{\beta_{j+2}}; \ldots\right],\ j = 0,1,\ldots,$$

$$\gamma_j = -\frac{\lambda_{j-1}((\lambda_{j-1} + (j-1)\nu)^2 + (j-1)\nu\mu)}{j(j+1)\nu^2\mu},\ j = 1,2,\ldots,$$

$$\beta_j = -\frac{j\nu((\lambda_j + \mu + j\nu)^2 + \mu(\lambda_{j-1} + \mu + j\nu))}{j(j+1)\nu^2\mu},\ j = 1,2,\ldots$$

In this work we also consider optimization problem for finding optimal strategy of control of the rate of input flow for system with limited number of retrials.

References

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